

On higher spin symmetries in de Sitter QFTs

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ABSTRACT: We consider the consequences of global higher-spin symmetries in quantum field theories on a fixed de Sitter background of spacetime dimension $D \geq 3$. These symmetries enhance the symmetry group associated with the isometries of the de Sitter background and thus strongly constrain the dynamics of the theory. In particular, we consider the case when a higher spin charge acts linearly on a scalar operator to leading order in a Fefferman-Graham expansion near the future/past conformal boundaries. We show that this implies that the expectation values of the operator inserted near the boundaries are asymptotically Gaussian. Thus, these operators have trivial cosmological spectra, and on global de Sitter these operators have only Gaussian correlations between operators inserted near future/past infinity. The latter result may be interpreted as an analogue of the Coleman-Mandula theorem for QFTs on de Sitter spacetime.

KEYWORDS: Higher Spin Symmetry, Integrable Field Theories

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1 Introduction

Quantum field theories (QFTs) are difficult to solve. This statement is already true for QFTs on Minkowski spacetime, but it becomes emphatically true for QFTs on curved spacetimes. Quantum field theory on curved backgrounds, including perturbative gravity, provides the current framework used to describe potentially observable quantum effects in cosmology [1]. Yet very few soluble QFTs exist in this setting, and perturbation theory remains the de facto technique to tackling such theories. The limited range of validity of standard perturbative techniques makes it challenging to investigate super-Hubble effects, such as those induced by fields with long-range order, which might be relevant to eternal inflation and the observed effective cosmological constant.

QFTs become more tractable when they include heightened symmetry – i.e. symmetries not realized in nature – whose presence provides additional handles with which we may grasp the theories. A classic example is higher spin (HS) symmetry. HS symmetries are symmetries whose generators transform has rank $n > 1$ tensors with respect to the spacetime isometry group. Typically, the presence of these symmetries in quantum theories is so constraining that at least some aspects of a theory may be solved exactly. In $D > 2$ dimensional Minkowski space, the Coleman-Mandula theorem [2] asserts that HS symmetries constrain the scattering matrix of a theory to be trivial. In $D = 2$ dimensions, the presence of such symmetries implies that scattering matrix has no particle production (particle number is conserved and scattering is elastic) [3]. When combined with conformal symmetry, HS symmetries yield interesting classes of soluble rational CFTs in 2D [4],

and in 3D CFTs they constrain the theory to have a free-field current algebra, and thus be essentially free [5]. In each of these cases, the presence of HS symmetries forces the underlying theory to behave as a free theory for at least a large set of physical observables.

In this paper we investigate consequences of HS symmetries in QFTs on a fixed de Sitter (dS) spacetime. de Sitter space provides the maximally symmetric cosmological model of an inflating spacetime, and is the lowest-order solution in the standard slow-roll expansion of inflation. The natural set of QFT observables we investigate are the vacuum expectation values of operators inserted near the future and past conformal boundaries. In the context of inflation, observables located near the future asymptotic boundary of de Sitter correspond to late-time expectation values which provide the input for cosmological power spectra. We also consider the correlation functions of observables located near both the past and future asymptotic boundaries of global de Sitter. These correlators describe the global dS analogue of a scattering experiment.¹ Our basic tactic is to analyze how the Ward identities associated with HS symmetries constrain the correlation functions of these observables. In many ways, our investigation is similar in spirit to recent cosmological “consistency conditions” and “soft theorems” (see, e.g., [7–11] and references therein), though we emphasize that our analysis does not include gravitational back-reaction.

More concretely, our analysis proceeds as follows. For simplicity we consider the effect of HS symmetries on correlations of a scalar operator $\phi(x)$. We assume the theory admits a charge $Q_p^{(s)}$ which is the spin $s > 1$ analogue of a translation in a dS Poincaré chart. Although this chart provides a convenient interpretation for $Q_p^{(s)}$, the charge is in fact well-defined everywhere on dS. The action of $Q_p^{(s)}$ on $\phi(x)$ is local and may be written as a sum of local operators $\mathcal{O}_A(x)$ of the schematic form

$$\left[Q_p^{(s)}, \phi(x)\right] = \sum_A C_A \mathcal{O}_A(x). \quad (1.1)$$

In general, any operator with the correct quantum numbers may appear on the right-hand side of this expression, making a general analysis of (1.1) intractable. However, if we are interested in correlators of $\phi(x)$ near the conformal boundaries of dS (i.e., near past/future asymptotic infinity), we may expand (1.1) in a Fefferman-Graham expansion in powers of conformal time η . Only those operators which scale with η in the same way as $\phi(x)$ will contribute to the commutator asymptotically. Thus, for instance, for a scalar with characteristic scaling $\phi(x) = O(\eta^\Delta)$, $\Delta > 0$, as $\eta \rightarrow 0$, we may truncate the right-hand side to operators which likewise scale like $O(\eta^\Delta)$ when evaluating the commutator at asymptotically late times ($\eta \rightarrow 0$).

Here we consider the simple case when $\phi(x)$ and its descendants are the only operators in the theory which scale like $O(\eta^\Delta)$ as $\eta \rightarrow 0$. In this case the action of $Q_p^{(s)}$ on $\phi(x)$ becomes *asymptotically linear* in $\phi(x)$ near the conformal boundary, i.e. the action takes

¹Here we mean only that these correlations measure the transition amplitude for states constructed at asymptotically early/late times. We will not attempt to establish a rigorous notion of asymptotic particle states for global dS. See [6] for a construction of such particle states and their corresponding S-matrix for *perturbatively* interacting QFTs on global de Sitter space.

the form

$$\left[Q_p^{(s)}, \phi(x) \right] \Big|_{O(\eta^\Delta)} = \mathcal{D}(x) \phi(x) \Big|_{O(\eta^\Delta)}, \quad (1.2)$$

where $\mathcal{D}(x)$ is a differential operator. When this occurs, we show that this implies that the leading $O(\eta^{n\Delta})$ behavior of an n -pt correlation function of $\phi(x)$ is Gaussian, i.e., it is composed of 2-pt correlations. The same conclusion holds in global dS for correlation functions in which each operator is placed near one of the asymptotic boundaries. Thus $\phi(x)$ has trivial cosmological spectra (no bispectrum, Gaussian trispectrum, etc.), and also has no “scattering” in global dS (when measured with respect to equivalent initial/final vacua).

The assumption that the action of a HS charge is asymptotically linear is clearly very restrictive. However, we regard this assumption as the appropriate dS analogue of one of the assumptions of the Coleman-Mandula theorem: a symmetry of the S-matrix is one which maps n -particle states to n -particle states [2]. Said differently, a symmetry-generating charge acts linearly on the field redefinition-invariant parts of the asymptotic correlation functions. In dS QFT, the leading asymptotic behavior of vacuum correlation functions near the conformal boundaries is, at least in perturbation theory, field redefinition-invariant and the key input in the perturbative de Sitter S-matrix [6]. Thus, we regard our result as providing an analogue of the Coleman-Mandula theorem for dS QFT.

We note in passing that HS symmetries on (asymptotically) de Sitter spacetimes have been of recent interest due to their roles in Vasiliev theories of HS gravity [12] as well as a potential realization of the de Sitter/Conformal Field Theory (dS/CFT) correspondence [13]. In general these theories include dynamical gravity which we do not consider here. However, when solutions to these theories may be described as having an exact de Sitter metric, as well as a matter sector which satisfies the properties stated in §2 below, then our results apply. The HS charges we consider here are more general than those which appear in this limit of Vasiliev theory, as the latter are generated by conserved traceless currents, whereas the conserved currents we consider need not be traceless.

This paper is organized as follows. After briefly reviewing necessary background material in §2, we analyze general aspects of HS symmetries in dS QFTs in §3. As a concrete and simple example, we provide a discussion of the HS symmetries present in dS complex Klein-Gordon theory in §4. In §5 we consider HS charges which act linearly on a scalar field everywhere on dS. Not surprisingly, when this occurs the correlation functions of the scalar field are constrained to be everywhere Gaussian. In §6 we consider the more general case of HS charges whose action becomes linear only asymptotically. We conclude with a brief discussion in §7.

2 de Sitter QFTs

We use this preliminary section to establish our notation as well as review background material relevant to our study.

We consider $D = d + 1$ dimensional de Sitter spacetime dS_D with curvature radius ℓ . The spacetime is conveniently described as a hyperboloid in a $D + 1$ -dimensional Minkowski

embedding space:

$$dS_D := \{X \in \mathbb{R}^{D,1} \mid X \cdot X = \ell^2\}. \quad (2.1)$$

This surface is preserved under the action of the embedding space Lorentz group $SO(D, 1)$, i.e. boosts and rotations in the embedding space which preserve the origin. This group is thus the isometry group of dS_D (the “dS group”). The entire manifold may be covered by the global coordinate chart

$$ds^2 = -d\tau^2 + \ell^2 \cosh^2(\tau/\ell) d\Omega_d^2, \quad (2.2)$$

where $d\Omega_d^2$ is the line element of the unit sphere S^d . This chart nicely displays the hyperboloid geometry of dS, and in particular the fact that the manifold may be foliated by compact Cauchy surfaces. The conformal boundary of the spacetime is composed of two disconnected components, future (past) conformal infinity $\mathcal{I}^{+(-)}$ located at $\tau \rightarrow +(-)\infty$, each conformal to S^d .

For our purposes, a more convenient set of coordinates is given by the (expanding) Poincaré chart:

$$ds^2 = \frac{\ell^2}{\eta^2} \left(-d\eta^2 + \delta_{ab} dx^a dx^b \right), \quad \eta \in (-\infty, 0), \quad x^a \in \mathbb{R}^d. \quad (2.3)$$

Here η is conformal time, roman indices run over spatial dimensions $1, \dots, D$, and δ_{ab} is the flat metric on \mathbb{R}^d . The expanding Poincaré chart covers only half of the dS manifold; the other half manifold may covered by a contracting Poincaré chart with conformal time $\eta \in (0, +\infty)$. In these coordinates the dS isometries may be described as: (i) translations and (ii) rotations on constant- η surfaces, (iii) the dilations $x^\mu \rightarrow \lambda x^\mu$, and (iv) special conformal transformations $x^\mu \rightarrow x^\mu + 2b^\nu x_\nu x^\mu - x_\nu x^\nu b^\mu$ generated by vectors b^μ tangent to constant- η surfaces.

With regard to any compact subset of spacetime, in the limit $\ell \rightarrow \infty$ the de Sitter geometry reduces to Minkowski spacetime. We refer to this limit, with all other dimensional quantities held fixed, as the flat-space limit. The Poincaré coordinates (2.3) are not well-suited for this limit; instead we use the “proper time” coordinate t :

$$\eta = -\ell e^{-t/\ell}, \quad t = -\ell \ln \left(-\frac{\eta}{\ell} \right), \quad (2.4)$$

so that the line element becomes

$$ds^2 = -dt^2 + e^{2t/\ell} \delta_{ab} dx^a dx^b, \quad t \in \mathbb{R}. \quad (2.5)$$

Taking $\ell \rightarrow \infty$ with these coordinates fixed, the line element indeed reduces to that of Minkowski space.

Given two points $X_i, X_j \in dS_D$ it is convenient to define the $SO(D, 1)$ -invariant *chordal distance*²

$$X_{ij} := \frac{1 - X_i \cdot X_j / \ell^2}{2}. \quad (2.6)$$

²This is actually one quarter the chordal distance.

Different causal relationships between the points are encoded in X_{ij} as follows:³

$$\begin{aligned} \text{spacelike separation:} \quad & X_{ij} > 0, \\ \text{null separation:} \quad & X_{ij} = 0, \\ \text{timelike separation:} \quad & X_{ij} < 0. \end{aligned} \tag{2.7}$$

The chordal distance may be expressed in Poincaré coordinates as

$$X_{ij} = \frac{|\vec{x}_i - \vec{x}_j|^2 - (\eta_i - \eta_j)^2}{4\eta_i\eta_j}. \tag{2.8}$$

We are interested in studying local dS QFTs which have standard properties of QFTs on curved spacetime (see e.g., [14, 15], as well as discussion in [16]). In particular, we restrict our attention to theories with the following properties:

- i) **dS covariance:** The theory does not select a preferred direction or otherwise spoil the unitary representation of the dS isometry group. For each dS Killing vector field (KVF) ξ^μ there exists an isometry generator in the QFT of the form

$$G_\xi := \int d\Sigma(x) n^\mu \xi^\nu T_{\mu\nu}(x) \Big|_\Sigma, \tag{2.9}$$

where Σ is a Cauchy surface, n^μ is the future-pointing unit normal vector, and $T_{\mu\nu}(x)$ is the QFT stress-tensor. The generators satisfy the $SO(D, 1)$ algebra inherited from the KVFs

$$[G_{\xi_1}, G_{\xi_2}] = -iG_{[\xi_1, \xi_2]}, \tag{2.10}$$

where for vector fields $[\xi_1, \xi_2]$ denotes the Lie bracket. The generators act on any quantum operator $\mathcal{O}(x)$ via a Lie derivative:

$$[G_\xi, \mathcal{O}(x)] = i\mathcal{L}_\xi \mathcal{O}(x). \tag{2.11}$$

- ii) **dS-invariant states:** The theory admits at least one state invariant under the action of the dS isometry group $SO(D, 1)$.⁴
- iii) **microlocal spectrum condition (“ μ SC”):** Correlation functions of the theory contain short-distance (ultraviolet) singularities consistent with the micro-local spectrum condition of [19]. Essentially, the μ SC states that the only singularities of correlation functions are at coincident configurations, and are of “positive frequency” as measured in any locally flat coordinate chart. This assures that the singularities present in correlation functions look like those of the usual Minkowski vacuum. The μ SC may be regarded as the generalization of the Hadamard condition to interacting theories [19–21].

³Spacelike-separated points in dS which may be connected by a geodesic satisfy $X_{ij} \in (0, 1]$.

⁴Not all theories satisfying (i) necessarily have such states. A well-known example is the massless, minimally-coupled Klein-Gordon field, when quantized via canonical quantization [17, 18].

- iv) **IR-regularity:** We assume that correlation functions of local operators do not grow too quickly as the chordal distance Z between two clusters of operators grows. For massive theories in dS, such correlation functions *decay* as the chordal separation Z increases [22–24]. For interacting massless theories, such correlation functions are known to grow in perturbation theory like a power of $(\log Z)$ (see [25, 26] for discussion of dS-invariant states, as well as [27–30] for less-symmetric states). To be concrete, we assume that the correlation functions of local operators clustered into two groups separated by chordal distance $|Z| \gg 1$ may be bounded by cZ for some finite constant c .⁵

Given these assumptions, we expect that correlation functions of scalar operators with respect to a dS-invariant state Ω admit representations as generalized Mellin-Barnes integrals which take the form [22–25, 32]:

$$\langle \phi(x_1) \dots \phi(x_n) \rangle_\Omega = \left[\prod_{i < j}^n \int_{C_{ij}} \frac{d\mu_{ij}}{2\pi i} X_{ij}^{\mu_{ij}} \right] \mathcal{M}(\vec{\mu}). \quad (2.12)$$

There is one integration variable μ_{ij} for each pairing x_i, x_j . Each μ_{ij} is integrated along a Mellin-Barnes contour C_{ij} , i.e. a contour traversed from $-i\infty$ to $+i\infty$ in the left half-plane, which may be diverted to left or right to avoid pole singularities in $\mathcal{M}(\vec{\mu})$. We refer to $\mathcal{M}(\vec{\mu}) := \mathcal{M}(\mu_{12}, \dots, \mu_{n-1,n})$ as the *Mellin amplitude* of the correlation function $\langle \phi(x_1) \dots \phi(x_n) \rangle_\Omega$. Mellin amplitudes contain pole singularities in the complex μ_{ij} planes. The contour integrals converge due to the fact that $\mathcal{M}(\vec{\mu})$ decays sufficiently fast – i.e. exponentially – as any $|\text{Im } \mu_{ij}| \rightarrow \infty$. For the generalized Mellin transforms to be defined at timelike separations one must add the standard position-space $i\epsilon$ prescriptions to the X_{ij} . These $i\epsilon$ prescriptions will be unimportant to our analysis so we will suppress them.

The details of these Mellin-Barnes representations will not be important to our analysis. The key point to take away is that these representations define functions of the chordal distances regarded as independent variables. The set of chordal distances which describe n points in dS are not all independent as they are constrained so that all n points “fit” in dS_D . However, the Mellin-Barnes integrals (2.12) define functions of the X_{ij} over a larger domain. Further details of the Mellin-Barnes representation are presented in [22, 23, 25].⁶

3 Higher spin symmetries on dS

The higher-spin symmetries we study are generated by a local, covariantly conserved current $J_{\mu_1 \dots \mu_s}(x)$ of rank $s > 2$. Given a rank- $(s+1)$ current and a rank s Killing tensor

⁵This assumption seems eminently reasonable for any theory which might be called massive or massless. However, in de Sitter there also exist unitary, tachyonic theories which violate this assumption. The simplest example is a discrete set of free, tachyonic scalars [31]. The correlation functions of these theories can grow like a power of Z . However, we know of no interacting theories which violate assumption (iv).

⁶Mellin transforms are a natural integral transform to consider whenever the underlying function has power-law asymptotics. As such, Mellin transforms are also useful tools in CFT and AdS/CFT (see, e.g., [33, 34] and references therein). Historically, Mellin transforms have played a key role in calculations of multi-loop Feynman diagrams in Minkowski space [35].

$K^{\nu_1 \dots \nu_s}(x)$ we may define a spin s conserved charge⁷

$$Q_K^{(s)} := \int d\Sigma(x) n^\mu K^{\nu_1 \dots \nu_s} J_{\mu\nu_1 \dots \nu_s}(x) \Big|_\Sigma. \quad (3.1)$$

Here Σ is a *global* dS Cauchy surface and n^μ is the future-pointing normal vector. Any de Sitter Killing tensor may be written as a product of KVF's [36], KVF's, i.e.,

$$K^{\nu_1 \dots \nu_s}(x) = \xi_1^{\nu_1} \dots \xi_s^{\nu_s}(x), \quad (3.2)$$

where each ξ_i^μ may be a distinct KVF. Using this form it is easy to show that the charges $Q_K^{(s)}$ and the dS generators G_ξ have the commutators

$$[G_\xi, Q_K^{(s)}] = -i Q_{\mathcal{L}_\xi K}^{(s)}. \quad (3.3)$$

Since ξ is a dS KVF, the action of the lie derivative \mathcal{L}_ξ on $K^{\nu_1 \dots \nu_s}(x)$ produces another Killing tensor of the form (3.2). Thus, the charges $Q_K^{(s)}$ and the dS generators G_ξ form a closed algebra which enlarges the $SO(D, 1)$ algebra of the generators alone.

We will be primarily concerned with the higher-spin charges

$$Q_p^{(s)} := \int d\Sigma(x) n^\mu p^{\nu_1} \dots p^{\nu_s} J_{\mu\nu_1 \dots \nu_s}(x) \Big|_\Sigma, \quad (3.4)$$

where p^μ is a KVF whose flow corresponds to translation along constant η surfaces in some Poincaré chart. Note that for a given p^μ the Poincaré time direction ∂_η is unique up to choice of sign (the sign corresponds to whether the Poincaré chart is expanding or contracting). It is convenient to adopt this expanding Poincaré chart to describe characteristics of $Q_p^{(s)}$. We emphasize, however, that $Q_p^{(s)}$ is defined over the global dS manifold. We normalize p^μ such that in this Poincaré chart $\delta_{ab} p^a p^b = 1$. Let us denote the dS generators in this chart by P_a (translations), D (dilations), R_{ab} (rotations), and K_a (SCTs). Then, for instance, $Q_p^{(1)}$ corresponds to a linear combination of the P_a 's. From (3.3) it follows that the $Q_p^{(s)}$ enjoy simple commutation relations with some of the dS generators:

$$[P_a, Q_p^{(s)}] = 0, \quad [D, Q_p^{(s)}] = -s Q_p^{(s)}, \quad [R^\perp, Q_p^{(s)}] = 0. \quad (3.5)$$

Here R^\perp represents the generators of rotations which preserves p^μ (these exist for $D \geq 4$). In general $Q_p^{(s)}$ are covariant under $SO(d)$ rotations in the sense that

$$R(\alpha) Q_p^{(s)} R^{-1}(\alpha) = Q_{R^{-1}p}^{(s)}. \quad (3.6)$$

On the other hand, the special conformal transformation generators K_a alter $Q_p^{(s)}$ in a complicated way, and $[K_a, Q_p^{(s)}]$ does not correspond to any $Q_p^{(s)}$.

The action of a HS charge $Q_p^{(s)}$ on a local operator $\mathcal{O}(x)$ is given by the commutator $[Q_p^{(s)}, \mathcal{O}(x)]$. For simplicity we will focus on the action of $Q_p^{(s)}$ on scalar operators. The most general action is given by the following:

⁷We do not require the current $J_{\mu\nu_1 \dots \nu_s}(x)$ nor the Killing tensor $K^{\nu_1 \dots \nu_s}(x)$ be traceless.

Lemma 3.1 *Let $\phi(x)$ be a local scalar operator. The most general form of the commutator $[Q_p^{(s)}, \phi(x)]$ is*

$$[Q_p^{(s)}, \phi(x)] = \sum_A \frac{1}{\eta^{s-k}} C_A^{\mu_1 \dots \mu_k} \mathcal{O}_{\mu_1 \dots \mu_k}^A(x). \quad (3.7)$$

Here A is a collective index labeling operators $\mathcal{O}_{\mu_1 \dots \mu_k}^A(x)$ which transform covariantly under $SO(D, 1)$, k is an integer (which depends on A) satisfying $0 \leq k \leq s$, and the $C_A^{\mu_1 \dots \mu_k}$ are constant coefficients. These coefficients are composed of products of p^μ and t^μ , where $t^\mu \partial_\mu = \partial_\eta$, such that there is an even (odd) number of p^μ 's when s is even (odd).

We prove this lemma in Appendix A. Note that in order for the dimensions to be consistent in (3.7), the operators $\mathcal{O}_{\mu_1 \dots \mu_k}^A(x)$ must have length dimension $1 - D/2 - k$. The explicit factors of η and t^μ are allowed in (3.7) because p^μ selects a unique time coordinate η .

Let us compare lemma 3.7 to the analogous result in Minkowski QFT. If a Minkowski theory has a HS current then one may construct, e.g., the HS charge

$$Q_1^{(s)} := \int d^d x J_{01\dots 1}(x) \Big|_{x^0=\text{const}}, \quad (3.8)$$

where we use standard Cartesian coordinates $\{x^0, x^1, \dots, x^d\}$. For $s > 1$ this is the higher spin analogue of a translation along x^1 . It is easy to show that in this case the action of $Q_1^{(s)}$ on a scalar field is of the form

$$[Q_1^{(s)}, \phi(x)] = \sum_A c_A \mathcal{O}_{1\dots 1}^A(x), \quad (3.9)$$

where c_A are constant coefficients. Comparing (3.9) to (3.7), we see that if a dS theory is to admit a smooth flatspace limit, it must be that terms involving t^μ in (3.7) must vanish as $\ell \rightarrow \infty$, either due to explicit factors of $1/\eta$ (which tend to zero in the limit), or because the operator vanishes in the limit.

Returning to the dS context, we assume that there exist dS-invariant states which are annihilated by $Q_p^{(s)}$. It follows that expectation values taken with respect to these states satisfy “charge conservation identities,” or Ward identities, obtained by commuting $Q_p^{(s)}$ through the string of operators. E.g., for such a state Ω we may commute $Q_p^{(s)}$ through $\langle \phi(x_1) \phi(x_2) \dots \phi(x_n) \rangle_\Omega$ to obtain

$$0 = \left\langle \left[Q_p^{(s)}, \phi(x_1) \right] \phi(x_2) \dots \phi(x_n) + \dots + \phi(x_1) \dots \phi(x_{n-1}) \left[Q_p^{(s)}, \phi(x_n) \right] \right\rangle_\Omega. \quad (3.10)$$

These Ward identities will be the central object of our study.

4 HS charges in free fields

In order to provide a concrete example of HS symmetries, in this section we review the HS symmetries present in complex Klein-Gordon theory on dS.

A massive, complex Klein-Gordon field on dS may be described by the classical action

$$S = \int d^D x \sqrt{-g} \left(-\nabla_\mu \phi^\dagger \nabla^\mu \phi(x) - M^2 \phi^\dagger \phi(x) \right), \quad M^2 > 0. \quad (4.1)$$

In general we let $M^2 = m^2 + \xi R(x)$, with $m^2 > 0$, ξ a coupling constant, and $R(x)$ the Ricci scalar which is constant on dS. The case $m^2 = 0$, $\xi = (d-1)/(4d)$ corresponds to a conformally-invariant theory; in terms of M^2 this “conformally coupled” mass is

$$M_{c.c.}^2 \ell^2 = \frac{d^2 - 1}{4}. \quad (4.2)$$

Upon quantization the theory possesses a unique dS-invariant state Ω satisfying the μ MC [17, 18]. The composite operators below are defined by normal ordering with respect to Ω .

The reader is undoubtedly familiar with the lowest-spin currents in the theory, namely the spin-1 “Klein-Gordon current” and the stress tensor:

$$J_\mu(x) = \phi^\dagger \overleftrightarrow{\nabla}_\mu \phi(x), \quad (4.3)$$

$$J_{\mu\nu}(x) = 2\nabla_{(\mu} \phi^\dagger \nabla_{\nu)} \phi(x) + 2\xi \nabla_\mu \nabla_\nu (\phi^\dagger \phi(x)) - 2\xi R_{\mu\nu} \phi^\dagger \phi(x) - g_{\mu\nu} (\nabla_\lambda \phi^\dagger \nabla^\lambda \phi(x) + 2\xi \square (\phi^\dagger \phi(x)) + M^2 \phi^\dagger \phi(x)), \quad (4.4)$$

where $R_{\mu\nu} = (d/\ell^2)g_{\mu\nu}$ is the dS Ricci tensor. Perhaps less familiar is the fact that the theory admits symmetric, covariantly conserved currents of every rank which are of the form

$$J_{\mu_1 \dots \mu_n}(x) = \sum_{j=0}^n c_j \nabla_{(\mu_1} \dots \nabla_{\mu_j} \phi^\dagger \nabla_{\mu_{j+1}} \dots \nabla_{\mu_n)} \phi(x) + \text{traces}, \quad (4.5)$$

where the c_j are constants and “traces” denote terms composed of partial traces of the terms written, multiplied by appropriate factors of the metric.⁸ The most tidy example is the spin-3 current, which with convenient normalization may be written

$$J_{\mu\nu\lambda}(x) = \frac{1}{4(d+2)} \left[(d-1) (\phi^\dagger \nabla_{(\mu} \nabla_\nu \nabla_{\lambda)} \phi(x) - \nabla_{(\mu} \nabla_\nu \nabla_{\lambda)} \phi^\dagger \phi(x)) - 3(3+d) \nabla_{(\mu} \phi^\dagger \overleftrightarrow{\nabla}_{\nu)} \nabla_{\lambda)} \phi(x) + 6g_{(\mu\nu} \nabla^\alpha \phi^\dagger \overleftrightarrow{\nabla}_{\lambda)} \nabla_\alpha \phi(x) + [6M^2 - (d-1)(3d+2)\ell^{-2}] g_{(\mu\nu} J_{\lambda)}(x) \right], \quad (4.6)$$

and which has trace

$$g^{\mu\nu} J_{\mu\nu\lambda}(x) = (M^2 - M_{c.c.}^2) J_\lambda(x). \quad (4.7)$$

Unfortunately, we are unaware of explicit expressions for these HS currents for general M^2 ; for the conformally coupled case expressions for the currents may be obtained from known CFT results (see, e.g., [37, 38]).

Let us examine the action of the resulting HS charges on $\phi(x)$. A straightforward if tedious way to compute the commutator $[Q_p^{(s)}, \phi(x)]$ is by direct application of the canonical commutation relations. Expressed at equal times in Poincaré coordinates, these familiar relations are

$$[\phi(\eta, \vec{x}), \pi(\eta, \vec{y})] = i \left(\frac{\eta}{\ell} \right)^d \delta^d(\vec{x}, \vec{y}), \quad (4.8)$$

⁸ In the classical field theory, the currents may also be made traceless when $M^2 = M_{c.c.}^2$. However, as in the familiar case of the stress tensor, we expect that this tracelessness may be spoiled in the quantum theory by anomalies due to the curved background. We thank E. Mottola for raising this point.

where $\pi(x)$ is the momentum conjugate to $\phi(x)$ and $\delta^d(\vec{x}, \vec{y})$ is the d -dimensional Dirac delta function. For example, for the spin-2 charge associated with the spin-3 current (4.6), diligent calculation yields

$$\left[Q_p^{(2)}, \phi(x)\right] = -i \frac{|\vec{p}|^2}{2(d+2)} \left(-\partial_\eta^2 + \frac{(d-1)}{\eta} \partial_\eta - \frac{M^2 \ell^2}{\eta^2} + \Delta_s \right) \phi(x) + i \partial_p^2 \phi(x). \quad (4.9)$$

Here $|\vec{p}|^2 = \delta_{ab} p^a p^b$, $\partial_p = p^\mu \partial_\mu$ is set to unity in the main text, and Δ_s is the Laplacian compatible with the flat metric on constant- η hypersurfaces. The terms in parenthesis are proportional to the KG wave operator and thus annihilate the field $\phi(x)$ on-shell. The final expression is then

$$\left[Q_p^{(2)}, \phi(x)\right] = i \partial_p^2 \phi(x). \quad (4.10)$$

For general s it is more efficient to use lemma 3.1 in order to prove that the commutator is

$$\left[Q_p^{(s)}, \phi(x)\right] = i \partial_p^s \phi(x). \quad (4.11)$$

The argument is as follows. Since the currents are bi-linear in $\phi(x)$, and since the canonical commutation relations map $\phi \times \phi \rightarrow \mathbb{C}$, it follows that the right-hand side of (4.11) must be linear in $\phi(x)$. The commutator is a solution to the Klein-Gordon equation, the thus right-hand side of (4.11) must also be a solution. The only term which is linear in $\phi(x)$, a solution to the Klein-Gordon equation, and consistent with lemma 3.1 is $\partial_p^s \phi(x)$.

5 HS charges with linear action

In this section we consider HS charges which act linearly on a scalar field $\phi(x)$. By this we mean that

$$\left[Q_p^{(s)}, \phi(x)\right] = \mathcal{D}(x) \phi(x), \quad (5.1)$$

where $\mathcal{D}(x)$ is a differential operator of the form

$$\mathcal{D}(x) = \sum_A \frac{1}{\eta^{s-k}} C_A^{\mu_1 \dots \mu_k} \mathcal{D}_{\mu_1 \dots \mu_k}^{(A)}, \quad (5.2)$$

where the coefficients $C_A^{\mu_1 \dots \mu_k}$ are as in lemma 3.1, and the $\mathcal{D}_{\mu_1 \dots \mu_k}^{(A)}$ are rank- k covariant differential operators composed of products of ∇_μ , $g_{\mu\nu}$, and \square . Within this set-up we shall prove the following:

Lemma 5.1 *Consider a QFT satisfying the properties of §2 in spacetime dimension $D \geq 3$. Let Ω be a dS -invariant state which is annihilated by the HS charge $Q_p^{(s)}$. Suppose that the action of $Q_p^{(s)}$ on a scalar field $\phi(x)$ is linear, and furthermore that $\mathcal{D}(x)$ contains the term $(p^\mu \partial_\mu)^s$. Then the correlation functions $\langle \phi(x_1) \dots \phi(x_n) \rangle_\Omega$ are Gaussian.*

Proof. Consider the Ward identity associated with commuting $Q_p^{(s)}$ through the correlation function

$$F := \langle \phi(x_1) \dots \phi(x_n) \rangle_\Omega, \quad (5.3)$$

where no pair of points is null-separated. We may regard F as a function of the $n(n-1)/2$ chordal distances X_{ij} . Due to the linear action (5.1) of $Q_p^{(s)}$, this Ward identity may be written as

$$0 = \sum_{k=1}^n \mathcal{D}(x_k) F. \quad (5.4)$$

Unpacking this expression results in several terms. Let us focus on terms generated by $[p^\mu(\partial/\partial x_1^\mu)]^s$. From the derivatives

$$\begin{aligned} \left(p^\mu \frac{\partial}{\partial x_1^\mu}\right) X_{12} &= \frac{\vec{p} \cdot (\vec{x}_1 - \vec{x}_2)}{2\eta_1 \eta_2} = - \left(p^\mu \frac{\partial}{\partial x_2^\mu}\right) X_{12}, \\ \left(p^\mu \frac{\partial}{\partial x_1^\mu}\right)^2 X_{12} &= \frac{p^2}{2\eta_1 \eta_2} = \left(p^\mu \frac{\partial}{\partial x_2^\mu}\right)^2 X_{12}, \end{aligned} \quad (5.5)$$

it follows that that (5.4) contains the terms

$$\begin{aligned} T_k &:= \left(\frac{1}{2\eta_1 \eta_2} \vec{p} \cdot (\vec{x}_1 - \vec{x}_2)\right)^{s-k} \left(\frac{1}{2\eta_1 \eta_3} \vec{p} \cdot (\vec{x}_1 - \vec{x}_3)\right)^k \left(\frac{\partial}{\partial X_{12}}\right)^{s-k} \left(\frac{\partial}{\partial X_{13}}\right)^k F, \\ &\text{for } k = 1, \dots, s-1. \end{aligned} \quad (5.6)$$

Each T_k depends on $\vec{p} \cdot (\vec{x}_1 - \vec{x}_2)$ and $\vec{p} \cdot (\vec{x}_1 - \vec{x}_3)$ in a distinct way which cannot arise from any other term in the Ward identity. In particular, these terms can only arise from the derivative operator $(p^\mu \partial/\partial x_1^\mu)^s$, and can only arise from one way of distributing the derivatives $\partial/\partial x_1^\mu$.⁹ Thus, each T_k must vanish individually in order for the Ward identity to be satisfied. The first line of (5.6) does not vanish for general configurations of points, and thus the factor on the second line must vanish. When $n > 3$ we may repeat this argument for terms of the form T_k but with x_2, x_3 replaced with other combinations of points in $\{x_2, \dots, x_n\}$. Ultimately we conclude that F must satisfy

$$\left(\frac{\partial}{\partial X_{1i}}\right)^{s-k} \left(\frac{\partial}{\partial X_{1j}}\right)^k F = 0, \quad i, j \in \{2, \dots, n\}, \quad k = 1, \dots, s-1. \quad (5.7)$$

We distinguish two ways the equalities (5.7) may be satisfied: i) F depends only on one chordal distance X_{1i} , or ii) F depends on more than one chordal distance X_{1i} , but must depend on each distance polynomially. In fact, the most general F is a sum of terms, each of which satisfies either (i) or (ii). However, possibility (ii) violates our assumption of IR regularity ((iv) in §2). If F depends polynomially on all chordal distances involving x_1 , then F grows polynomially in the chordal distance as x_1 is taken to large mutual chordal separation from the remaining points. Thus we conclude that F must be a sum of terms, each of which depends on only one chordal distance X_{1i} .

We can now repeat the argument for those terms in the Ward identity which are generated by $(p^\mu \partial/\partial x_j^\mu)^s$, $j = 2, \dots, n$, and which yield constraints similar to (5.7) but

⁹The case $D = 2$ is different. In this case there is only one spatial dimension, so $p^\mu \partial_\mu$ does not have the same effect of selecting a preferred spatial direction. One may produce terms with the same coordinate dependence of T_k by acting with time derivatives ∂_{η_1} . As a result, it is no longer the case that the T_k must vanish.

with x_1 swapped for another point. Eventually we are led to conclude that F is a sum of terms, each of which depends on only one chordal distance per spacetime point. Thus F is Gaussian. This concludes the proof. ■

6 HS charges with asymptotically linear action

Next we consider HS charges with a less restrictive form of action on scalar operators. Here we consider actions which become linear only in the neighborhood of the asymptotic boundaries. As operator expressions, the commutator $[Q_p^{(s)}, \phi(x)]$ and Ward identity may be expanded as a Laurent expansion with respect to conformal time η (or the coordinate q defined below) in the spirit of a Fefferman-Graham expansion. For simplicity we focus on the case where $\phi(x)$ is a scalar operator with characteristic leading behavior near the conformal boundary

$$\phi(x) = O(\eta^\Delta), \quad \Delta > 0, \quad \text{as } \eta \rightarrow 0. \quad (6.1)$$

Scalars with this asymptotic form, and with $0 < \Delta < d/2$, are often referred to as “light” fields, because in the canonical example of a Klein-Gordon theory such operators correspond to fields with mass of order ℓ^{-2} . Our results below are valid for any positive Δ .

Definition 6.1 *Let $\phi(x)$ be a scalar operator on dS_D with characteristic scaling $\phi(x) = O(\eta^\Delta)$, $\Delta > 0$, as $\eta \rightarrow 0$. Then the action of charge Q on $\phi(x)$ is asymptotically linear if the leading term in the commutator takes the form*

$$[Q, \phi(x)] \Big|_{O(\eta^\Delta)} = \mathcal{D}(x)\phi(x) \Big|_{O(\eta^\Delta)}, \quad (6.2)$$

where $\mathcal{D}(x)$ is a differential operator of the form described in lemma 5.1.

Theorem 6.2 *Let $\phi(x)$ be a scalar operator on dS_D , $D \geq 3$, with characteristic scaling $\phi(x) = O(\eta^\Delta)$, $\Delta > 0$, as $\eta \rightarrow 0$, and let Ω be a dS -invariant state annihilated by the HS charge $Q_p^{(s)}$. If the action of $Q_p^{(s)}$ on $\phi(x)$ is asymptotically linear and contains the term $(p^\mu \partial_\mu)^s$, then the leading $O(\eta^{n\Delta})$ behavior of the equal-time correlation functions $\langle \phi(\eta, \vec{x}_1) \dots \phi(\eta, \vec{x}_n) \rangle_\Omega$ is Gaussian.*

Proof. After a bit of simplification the proof is very similar to that of lemma 5.1. Let $F = \langle \phi(\eta, \vec{x}_1) \dots \phi(\eta, \vec{x}_n) \rangle_\Omega$ be an equal-time correlation function evaluated at n non-coincident points. The Ward identity now implies that as $\eta \rightarrow 0$

$$\sum_{k=1}^n \mathcal{D}(x_k) F = O(\eta^{n\Delta+\epsilon}), \quad \epsilon > 0. \quad (6.3)$$

Once again we focus on the terms arising from the derivative $(p^\mu \partial / \partial x_1^\mu)^s$ within $\mathcal{D}(x_1)$. This yields terms of the form T_k as in (5.6). Due to their unique dependence on $\vec{p} \cdot (\vec{x}_1 - \vec{x}_2)$ and $\vec{p} \cdot (\vec{x}_1 - \vec{x}_3)$, each of these terms must satisfy the fall-off condition individually, i.e.

each must be $O(\eta^{n\Delta+\epsilon})$. Following through the same logic as in the previous proof, we quickly conclude that

$$\left(\frac{\partial}{\partial X_{1i}}\right)^{s-k} \left(\frac{\partial}{\partial X_{1j}}\right)^k F = O(\eta^{n\Delta+\epsilon}), \quad i, j \in \{2, \dots, n\}, \quad k = 1, \dots, s-1. \quad (6.4)$$

To proceed further let $x_{ij} := |\vec{x}_i - \vec{x}_j|^2/4$ so that

$$X_{ij} = \frac{x_{ij}}{\eta^2}; \quad (6.5)$$

then (6.4) may be written

$$\left(\frac{\partial}{\partial x_{1i}}\right)^{s-k} \left(\frac{\partial}{\partial x_{1j}}\right)^k F = O(\eta^{n\Delta-2s+\epsilon}), \quad i, j \in \{2, \dots, n\}, \quad k = 1, \dots, s-1. \quad (6.6)$$

We next expand F in a Laurent expansion with respect to η . The leading term is

$$F = \langle \phi(\eta, \vec{x}_1) \dots \phi(\eta, \vec{x}_n) \rangle_\Omega \Big|_{O(\eta^{n\Delta})} =: \eta^{n\Delta} f, \quad (6.7)$$

where f is a function of the x_{ij} but does not depend on η . We may write f explicitly in terms of Mellin amplitude $\mathcal{M}(\vec{\mu})$ of F as

$$f := \left[\prod_{i < j}^n \int'_{C_{ij}} \frac{d\mu_{ij}}{2\pi i} x_{ij}^{\mu_{ij}} \right] \mathcal{M}(\vec{\mu}), \quad (6.8)$$

where the prime on the integrals denotes that contours are traversed such that $(\sum_{i < j}^n \mu_{ij} + \frac{n\Delta}{2}) = 0$.¹⁰ The key point is that f may be regarded as a function of independent variables x_{ij} . It follows that f satisfies

$$\left(\frac{\partial}{\partial x_{1i}}\right)^{s-k} \left(\frac{\partial}{\partial x_{1j}}\right)^k f = 0, \quad k = 1, \dots, s-1. \quad (6.9)$$

The equation (6.9) is the same as (5.7). Thus the remainder of this proof mimics that of the previous section. In order for f to satisfy (6.9) it must be either Gaussian with respect to the spatial coordinates \vec{x}_i , or it must be polynomial in the distances x_{1i} . But here the polynomial form is ruled out by the simple fact that dS-invariance of the correlator demands that f behave under a dilation as

$$f(\lambda \vec{x}_1, \dots, \lambda \vec{x}_n) = \lambda^{-n\Delta} f(\vec{x}_1, \dots, \vec{x}_n). \quad (6.10)$$

Thus f is Gaussian. ■

¹⁰There will be singularities in $\mathcal{M}(\vec{\mu})$ at points along this set of contours, so (6.8) includes both residue and principal parts.

We can quickly extend this result to the case where operators are inserted in the neighborhood of both asymptotic boundaries of global dS. It is convenient to switch time coordinate to $q = -\ell^2/\eta$, yielding the line element

$$ds^2 = -\frac{\ell^2}{q^2}dq^2 + \frac{q^2}{\ell^2}d\vec{x}^2, \quad q \in \mathbb{R}. \quad (6.11)$$

This coordinate chart covers all of dS_D . In this chart the conformal boundary is composed of two copies of \mathbb{R}^d located at $|q| \rightarrow \infty$, plus two points located at $q = 0$, $|\vec{x}| \rightarrow \infty$.

Theorem 6.3 *Let $\phi(x)$ be a scalar operator on dS_D , $D \geq 3$, with characteristic scaling $\phi(x) = O(q^{-\Delta})$, $\Delta > 0$, as $|q| \rightarrow \infty$, and let Ω be a dS-invariant state annihilated by the HS charge $Q_p^{(s)}$. Let the action of $Q_p^{(s)}$ on $\phi(x)$ be asymptotically linear as $|q| \rightarrow \infty$ and contain the term $(p^\mu \partial_\mu)^s$. Consider the correlation function $\langle \phi(q_1, \vec{x}_1) \dots \phi(q_n, \vec{x}_n) \rangle_\Omega$, with each q_1, \dots, q_n equal to $\pm q$, evaluated at non-null separations as $|q| \rightarrow \infty$. The leading $O(q^{-n\Delta})$ behavior of this correlation function is Gaussian.*

The proof is essentially the same as for the previous theorem. As for the case of a single boundary, one considers points such that no $x_{ij} = 0$. This assures that the points x_i and x_j are not coincident (when $q_i = q_j$) or null-separated (when $q_i = -q_j$) as $|q| \rightarrow \infty$.

7 Discussion

In this work we have examined the constraints imposed by the presence of higher spin symmetries in dS QFTs. Our main result was to show that if a HS charge acts linearly on a scalar operator near the asymptotic conformal boundaries, then the vacuum expectation values of that operator are asymptotically Gaussian. Thus the cosmological spectra associated with the scalar field are trivial, as is “scattering” in global dS. The condition that a HS charge have asymptotically linear action is analogous to the stipulation of the Coleman-Mandula theorem that symmetries of the S-matrix map n -particle states to n -particle states. Thus, we regard our result as a de Sitter analogue of the Coleman-Mandula theorem, specialized to the case of HS symmetries.

Although for simplicity we have focused our discussion on scalar operators with real weight $\Delta > 0$, we expect quite analogous results to hold for spinor and tensor operators, as well as scalar operators with complex weights. It would be interesting to consider vacuum states which do not satisfy the microlocal spectrum condition (i.e., de Sitter “ α -vacua” [17, 18]). At the level of linear theories, these states are known to have interesting scattering properties [39]; they may also admit novel interpretations in dS/CFT [40]. Our results rely crucially on the dilation and SCT symmetries of dS. It would therefore also be interesting to examine theories with HS symmetry on backgrounds for which the dS symmetry is slightly broken by non-zero slow-roll parameters.

Finally, we note that our main results, theorems 6.2 and 6.3, require spacetime dimension $D > 2$. In 2D Minkowski space there exist a rich class of non-conformal QFTs with HS symmetry and integrable scattering matrices (see e.g., [41–44]). It would be very

interesting to examine these theories on a dS background. It is possible that the HS symmetries, if they survive the relocation to dS, could provide integrable structures and allow exact results analogues to the Minkowski S-matrices. We leave this for future study.

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A Proof of lemma 3.1

To begin we write the general form of the commutator of $\phi(x)$ with a HS charge,

$$\left[Q_K^{(s)}, \phi(x)\right] = \sum_A \tilde{C}_A^{\mu_1 \dots \mu_k}(x) \mathcal{O}_{\mu_1 \dots \mu_k}^A(x), \quad (\text{A.1})$$

where the $\tilde{C}_A^{\mu_1 \dots \mu_k}(x)$ are coefficient functions which are taken to have length dimension $k - s$; in order to make the dimensionality of this equation consistent, the operators $\mathcal{O}_{\mu_1 \dots \mu_s}^A(x)$ must have length dimension $1 - D/2 - k$. Consider the action of a dS generator on this commutator. From the Jacobi identity we obtain

$$\left[G_\xi, \left[Q_K^{(s)}, \phi(x)\right]\right] = \left[\left[G_\xi, Q_K^{(s)}\right], \phi(x)\right] + \left[Q_K^{(s)}, \left[G_\xi, \phi(x)\right]\right]. \quad (\text{A.2})$$

We say that G_ξ preserves $Q_K^{(s)}$ when

$$\left[G_\xi, Q_K^{(s)}\right] = \epsilon Q_K^{(s)}, \quad (\text{A.3})$$

for some constant ϵ . In this case we obtain from (A.2)

$$\left[G_\xi, \left[Q_K^{(s)}, \phi(x)\right]\right] = (i\mathcal{L}_\xi + \epsilon) \left[Q_K^{(s)}, \phi(x)\right]. \quad (\text{A.4})$$

On the other hand, from the ansatz (A.1) it follows that

$$\left[G_\xi, \left[Q_K^{(s)}, \phi(x)\right]\right] = \sum_A \tilde{C}_A^{\mu_1 \dots \mu_k}(x) i\mathcal{L}_\xi \mathcal{O}_{\mu_1 \dots \mu_k}^A(x). \quad (\text{A.5})$$

Taking the difference of these equations we obtain a constraint on the coefficient functions:

$$(i\mathcal{L}_\xi + \epsilon) \tilde{C}_A^{\mu_1 \dots \mu_k}(x) = 0. \quad (\text{A.6})$$

We now apply this result to $Q_p^{(s)}$. There are several dS generators which preserve this charge. First consider the translation generators P_a for which $\epsilon = 0$. It follows from (A.6) that the $\tilde{C}_A^{\mu_1 \dots \mu_k}(x)$ cannot depend on the spatial variables x^a . The generator of dilations D also preserves $Q_p^{(s)}$, with $\epsilon = is$. In this case (A.6) requires that non-zero components of $\tilde{C}_A^{\mu_1 \dots \mu_k}(\eta)$ be $O(\eta^{k-s})$. For $D \geq 4$ there exist rotations which leave p^μ unchanged; the associated generators R^\perp preserve $Q_p^{(s)}$ with $\epsilon = 0$. The existence of these generators implies that $\tilde{C}_A^{\mu_1 \dots \mu_k}$ are composed of tensors invariant under the $SO(d-1)$ rotations which preserve p^μ .

We may also consider the discrete parity transformations of \mathbb{R}^d on the constant η surfaces. Those that preserve p^μ imply that, in all dimensions, the $\tilde{C}_A^{\mu_1 \dots \mu_k}(\eta)$ are equal to η^{k-s} times a constant tensor composed of δ^{ab} , p^μ , and $t^\mu = \delta_\eta^\mu$. The discrete transformation which acts as $p^\mu \rightarrow -p^\mu$ further requires the \tilde{C}_A to have an even (odd) number of p^μ 's when s is even (odd). Furthermore, any factor of δ^{ab} in $\tilde{C}_A^{\mu_1 \dots \mu_k}(\eta)$ may be re-cast as factor of the inverse metric $g^{\mu\nu}$, and this may be absorbed into the definition of the relevant operator.

Bringing everything together, we conclude that we may write the coefficients as

$$\tilde{C}_A^{\mu_1 \dots \mu_k}(\eta) = \frac{1}{\eta^{s-k}} C_A^{\mu_1 \dots \mu_k}, \quad (\text{A.7})$$

where the $C_A^{\mu_1 \dots \mu_k}$ are constant coefficients composed of factors of p^μ and t^μ , with the additional requirement of s modulo 2 factors of p^μ . Up to this point we have proven the form (3.7), except that we have yet to limit the range of k .

Next we consider those dS generators which do not preserve $Q_p^{(s)}$. In this case the action of a generator on a higher-spin charge produces a new charge,

$$\left[G_\xi, Q_{K_1}^{(s)} \right] = \kappa Q_{K_2}^{(s)}, \quad (\text{A.8})$$

for some constant κ . By (3.3) $K_2^{\mu_1 \dots \mu_s} \propto \mathcal{L}_\xi K_1^{\mu_1 \dots \mu_s}$. We may once again use the Jacobi identity to obtain a constraint satisfied by the coefficients functions, though now this constraint involves the coefficient functions corresponding to two HS charges. For the case (A.8) we obtain

$$-i\mathcal{L}_\xi C_{A,K_1}^{\mu_1 \dots \mu_k}(x) = \kappa C_{A,K_2}^{\mu_1 \dots \mu_k}(x). \quad (\text{A.9})$$

In order to use (A.9), let us consider without loss of generality case $p^\mu \partial_\mu = \partial_1$, and let s_1^μ be the KVF associated with the special conformal transformation with parameter $b^\mu \propto p^\mu$. It follows from the $SO(D, 1)$ algebra satisfied by the KVFs that

$$(\mathcal{L}_{s_1})^{2s+1} (p^{\mu_1} \dots p^{\mu_s}) = 0. \quad (\text{A.10})$$

Denoting the SCT generator associated with s_1^μ by K_1 it then follows that

$$\underbrace{\left[K_1, \left[K_1, \dots \left[K_1, Q_p^{(s)} \right] \dots \right] \right]}_{2s+1} = 0. \quad (\text{A.11})$$

Combining this result with (A.8) we obtain the following constraint on the coefficient functions:

$$(\mathcal{L}_{s_1})^{2s+1} \left(\eta^{k-s} C_A^{\mu_1 \dots \mu_k} \right) = 0. \quad (\text{A.12})$$

Given the form of $C_A^{\mu_1 \dots \mu_k}$ this equation is satisfied only for $k \leq s$. To see this we first note

$$\mathcal{L}_{s_1}^3 \eta^{-1} = 0, \quad \mathcal{L}_{s_1}^3 p^\mu = 0, \quad \mathcal{L}_{s_1}^3 t^\mu = 0, \quad (\text{A.13})$$

from which it follows that (A.12) is satisfied for $0 \leq k \leq s$. However, \mathcal{L}_{s_1} does not annihilate positive powers of η , and thus (A.12) is not satisfied for $k > s$. This proves the lemma. ■

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